

Indian Statistical Institute, Bangalore
B. Math (III)

Second Semester 2016-2017

Mid-Semester Examination : Statistics (IV)

Date: 17-02-2017

Maximum Score 40

Duration: 3 Hours

1. (a) Let X and Y be two random variables such that X is stochastically larger than Y . If there exists a constant c such that $\Pr [c(X - Y) < 0] = 1$ then what can you say about the constant c and the support of X in comparison to the support of Y ?
- (b) Develop a test, explaining the set-up, for the testing of hypothesis problem for κ_p , the p th quantile, $0 < p < 1$, $H_0 : \kappa_p = \kappa_p^0$ against $H_1 : \kappa_p > \kappa_p^0$, κ_p^0 being a specified constant.

[4 + 8 = 12]

2. (a) Let X be a random variable that is symmetrically distributed about c . Let Y be another random variable such that $X + Y = 2c$. What can you say about the distribution of Y ?
- (b) Develop the Sign test and obtain $100(1 - \alpha)\%$ confidence interval for the median.

[4 + 8 = 12]

3. (a) Let X and Y be two independent random variables drawn from the distribution given by

Value	a_1	a_2	\cdots	a_k
Probability	p_1	p_2	\cdots	p_k

where $p_j > 0$ and $\sum_{j=1}^k p_j = 1$.

Let further U be a random variable with distribution given by

Value	p_1	p_2	\cdots	p_k
Probability	$\frac{1}{k}$	$\frac{1}{k}$	\cdots	$\frac{1}{k}$

Show that

$$\Pr [X \text{ and } Y \text{ are tied}] = k \text{Var}(U) + \frac{1}{k}.$$

- (b) Develop the Wald-Wolfowitz runs test and explain how would you tackle the problem of ties.

[4 + 8 = 12]

4. (a) Define linear rank statistics.
- (b) Develop the Wilcoxon Rank Sum Test.

[4 + 8 = 12]